ЗАДАЧИ ОД ГРАНИЧНА ВРЕДНОСТ НА НИЗА (2)

1 а) $\lim\limits_{n\to\infty}\frac{6n^3+3n-4}{1-2n^3} = \lim\limits_{n\to\infty}\frac{6n^3+3n-4}{1-2n^3}\ \begin{matrix}/:n^3\\ /:n^3\end{matrix} = \lim\limits_{n\to\infty}\frac{6+\frac{3}{n^2}-\frac{4}{n^3}}{\frac{1}{n^3}-2} = -3$

б) $\lim\limits_{n\to\infty}\frac{(n+2)^2}{1-2n^2} = \left(\text{скратен формула } (A+B)^2 = A^2+2AB+B^2\right) = \lim\limits_{n\to\infty}\frac{n^2+4n+4}{1-2n^2}\ /:n^2 =$

$= \lim\limits_{n\to\infty}\frac{1+\frac{4}{n}+\frac{4}{n^2}}{\frac{1}{n^2}-2} = -\frac{1}{2}$

2 а) $\lim\limits_{n\to\infty}\frac{\sqrt[3]{8n^3+3n+1}}{5n-2}\ \begin{matrix}/:n\\ /:n\end{matrix} = \lim\limits_{n\to\infty}\frac{\frac{\sqrt[3]{8n^3+3n+1}}{n}}{\frac{5n}{n}-\frac{2}{n}} = \left(\begin{matrix}n \text{ го внесуваме во}\\ \text{коренот и станува}\\ n^3\end{matrix}\right)$

$= \lim\limits_{n\to\infty}\frac{\sqrt[3]{\frac{8n^3}{n^3}+\frac{3n}{n^3}+\frac{1}{n^3}}}{5-\frac{2}{n}} = \lim\limits_{n\to\infty}\frac{\sqrt[3]{8+\frac{3}{n}+\frac{1}{n^3}}}{5-\frac{2}{n}} = \frac{\sqrt[3]{8}}{5} = \frac{2}{5}$

б) $\lim\limits_{n\to\infty}\frac{\sqrt{16n^2-5n+2}}{\sqrt[3]{27n^3-6}}\ \begin{matrix}/:n\\ /:n\end{matrix} = \lim\limits_{n\to\infty}\frac{\frac{\sqrt{16n^2-5n+2}}{n}}{\frac{\sqrt[3]{27n^3-6}}{n}} = \left(\begin{matrix}n \text{ кога се внесува}\\ \text{во } \sqrt{} \text{ станува } n^2,\\ \text{а кога се внесува}\\ \text{во } \sqrt[3]{} \text{ станува } n^3\end{matrix}\right.$

$\left.\text{Те ойшто важи } n^p\sqrt[q]{x} = \sqrt[q]{n^{p\cdot q}x}\right)$

$= \lim\limits_{n\to\infty}\frac{\sqrt{\frac{16n^2}{n^2}-\frac{5n}{n^2}+\frac{2}{n^2}}}{\sqrt[3]{\frac{27n^3}{n^3}-\frac{6}{n^3}}} = \lim\limits_{n\to\infty}\frac{\sqrt{16-\frac{5}{n}+\frac{2}{n^2}}}{\sqrt[3]{27-\frac{6}{n^3}}} = \frac{\sqrt{16}}{\sqrt[3]{27}} > \frac{4}{3}$

3. а) $\lim\limits_{x\to\infty}\frac{3\cdot 5^{x+1}+6^{x+2}}{5^x-6^{x-1}} = \lim\limits_{n\to\infty}\frac{3\cdot 5^n\cdot 5+6^n\cdot 36}{5^n-\frac{6^n}{6}}\ \begin{matrix}/:6^n\\ /:6^n\end{matrix} = \lim\limits_{n\to\infty}\frac{15\cdot\left(\frac{5}{6}\right)^n+36}{\left(\frac{5}{6}\right)^n-\frac{1}{6}} = \frac{36}{-\frac{1}{6}} = -216$

б) $\lim\limits_{x\to\infty}\frac{1-3^{x+1}}{2+3^{x-1}} = \lim\limits_{n\to\infty}\frac{1-3^n\cdot 3}{2+\frac{3^n}{3}}\ /:3^n = \lim\limits_{n\to\infty}\frac{\frac{1}{3^n}-3}{\frac{2}{3^n}+\frac{1}{3}} = \frac{-3}{\frac{1}{3}} = -9$

4. а) $\lim\limits_{x\to\infty}\frac{n!+(n+1)!}{(n+2)!}$

Поищем base: $5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$, $4! = 4\cdot 3\cdot 2\cdot 1$, $3! = 3\cdot 2\cdot 1$

т.е. $5! = 5\cdot\underbrace{4\cdot 3\cdot 2\cdot 1}_{4!} = 5\cdot 4!$

или $5! = 5\cdot 4\cdot\underbrace{3\cdot 2\cdot 1}_{3!} = 5\cdot 4\cdot 3!$

т.е. $n! = n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1$

$(n+1)! = (n+1)\cdot n\cdot(n-1)\cdots 2\cdot 1$

Значит $(n+1)! = (n+1)\underbrace{n(n-1)\cdots 2\cdot 1}_{n!} = (n+1)\cdot n!$

и $(n+2)! = (n+2)\cdot(n+1)\cdot\underbrace{n\cdot(n-1)\cdots 2\cdot 1}_{n!} = (n+2)(n+1)\cdot n!$

$\lim\limits_{x\to\infty}\frac{n!+(n+1)!}{(n+2)!} = \lim\limits_{n\to\infty}\frac{n!+(n+1)\cdot n!}{(n+2)(n+1)\cdot n!} = \lim\limits_{n\to\infty}\frac{n!\left[1+(n+1)\right]}{(n+2)(n+1)\cdot n!} =$

$= \lim\limits_{n\to\infty}\frac{n+2}{(n+2)(n+1)} = \lim\limits_{n\to\infty}\frac{1}{n+1} = 0$

б) $\lim\limits_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}=\lim\limits_{n\to\infty}\frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}=\lim\limits_{n\to\infty}\frac{(n+1)![n+2+1]}{(n+1)![n+2-1]}=$

$=\lim\limits_{n\to\infty}\frac{n+3\ \ /:n}{n+1\ \ /:n}=\lim\limits_{n\to\infty}\frac{1+\frac{3}{n}}{1+\frac{1}{n}}=1$ ●

$\frac{3}{n}\to 0$, $\frac{1}{n}\to 0$

в) $\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n\cdot(n+2)}$

Броителят е збир на първите n членове на АП $1,2,3,\ldots,n$ те на АП со

$a_1=1$

$d=1$, па зашто

$S_n=\frac{n}{2}\left(2a_1+(n-1)d\right)$

$S_n=\frac{n}{2}\left(2\cdot 1+(n-1)\cdot 1\right)$

$S_n=\frac{n}{2}\left(2+n-1\right)$

$S_n=\frac{n}{2}(n+1)$ те $1+2+3+\ldots+n=\frac{n}{2}(n+1)$

$\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n\cdot(n+2)}=\lim\limits_{n\to\infty}\frac{\frac{n}{2}(n+1)\ \ /:n}{n(n+2)\ \ /:n}=\lim\limits_{n\to\infty}\frac{\frac{1}{2}\left(1+\frac{1}{n}\right)}{1+\frac{2}{n}}=\frac{1}{2}$ ●

$\frac{1}{n}\to 0$, $\frac{2}{n}\to 0$

б) $\lim\limits_{n\to\infty}\frac{1+3+5+\ldots+2n-1}{(2n+1)\cdot(2n-1)}$

Броителят е збир на първите n членове на АП $1, 3, 5, \ldots 2n-1$ те на АП со

$a_1 = 1$

$d = 2$, па зајдоа

$S_n = \frac{n}{2}\left(2a_1+(n-1)d\right)$

$S_n = \frac{n}{2}\left(2\cdot 1+(n-1)\cdot 2\right)$

$S_n = \frac{n}{2}\left(2+2n-2\right)$

$S_n = \frac{n}{2}\cdot 2n$

$S_n = n^2$ те $1+3+5+\ldots+2n-1 = n^2$

$$\lim_{n\to\infty}\frac{1+3+5+\ldots+2n-1}{(2n+1)\cdot(2n-1)} = \lim_{n\to\infty}\frac{n^2}{4n^2-1}\ \Big/:n^2 = \lim_{n\to\infty}\frac{1}{4-\frac{1}{n^2}} = \frac{1}{4}$$

$(A+B)\cdot(A-B)$; A^2-B^2 ; $\frac{1}{n^2}\to 0$

6а) $\lim\limits_{n\to\infty}\left(n-\sqrt{n^2-3n+8}\right)$

Имаме ор облик $\infty-\infty$, што е неопределен израз, па зајдоа дадената разлика $n-\sqrt{n^2-3n+8}$, ќе ја множиме $\frac{n+\sqrt{n^2-3n+8}}{n+\sqrt{n^2-3n+8}}$, со цел да се примени формата $(A-B)\cdot(A+B)=A^2-B^2$.

$$\lim_{n\to\infty}\left(n-\sqrt{n^2-3n+8}\right) = \lim_{n\to\infty}\left(n-\sqrt{n^2-3n+8}\right)\cdot\frac{n+\sqrt{n^2-3n+8}}{n+\sqrt{n^2-3n+8}} =$$

$$= \lim_{n\to\infty}\frac{n^2-\left(\sqrt{n^2-3n+8}\right)^2}{n+\sqrt{n^2-3n+8}} = \lim_{n\to\infty}\frac{n^2-n^2+3n-8}{n+\sqrt{n^2-3n+8}}\ \Big/:n =$$

$$= \lim_{n\to\infty}\frac{3-\frac{8}{n}}{1+\sqrt{1-\frac{3}{n}+\frac{8}{n^2}}} = 3$$

$\frac{8}{n}\to 0$; $\frac{3}{n}\to 0$; $\frac{8}{n^2}\to 0$

б) $\lim\limits_{n\to\infty}\left(\sqrt{n^2+3n}-\sqrt{n^2+2n}\right)=\lim\limits_{n\to\infty}\left(\sqrt{n^2+3n}-\sqrt{n^2+2n}\right)\cdot\dfrac{\sqrt{n^2+3n}+\sqrt{n^2+2n}}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=$

$=\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{n^2+3n}\right)^2-\left(\sqrt{n^2+2n}\right)^2}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=\lim\limits_{n\to\infty}\dfrac{n^2+3n-n^2-2n\ /:n}{\sqrt{n^2+3n}+\sqrt{n^2+2n}\ /:n}=$

$=\lim\limits_{n\to\infty}\dfrac{3-2}{\sqrt{1+\frac{3}{n}}+\sqrt{1+\frac{2}{n}}}=\dfrac{1}{2}$ ●

($\frac{3}{n}\to 0$, $\frac{2}{n}\to 0$)

в) $\lim\limits_{n\to\infty}\left(\sqrt{5n^2+4n+2}-\sqrt{5n^2-2n-1}\right)=$

$=\lim\limits_{n\to\infty}\left(\sqrt{5n^2+4n+2}-\sqrt{5n^2-2n-1}\right)\cdot\dfrac{\sqrt{5n^2+4n+2}+\sqrt{5n^2-2n-1}}{\sqrt{5n^2+4n+2}+\sqrt{5n^2-2n-1}}=$

$=\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{5n^2+4n+2}\right)^2-\left(\sqrt{5n^2-2n-1}\right)^2}{\sqrt{5n^2+4n+2}+\sqrt{5n^2-2n-1}}=\lim\limits_{n\to\infty}\dfrac{5n^2+4n+2-5n^2+2n+1\ /:n}{\sqrt{5n^2+4n+2}+\sqrt{5n^2-2n-1}\ /:n}$

$=\lim\limits_{n\to\infty}\dfrac{6+\frac{3}{n}}{\sqrt{5+\frac{4}{n}+\frac{2}{n^2}}+\sqrt{5-\frac{2}{n}-\frac{1}{n^2}}}=\dfrac{6}{2\sqrt{5}}=\dfrac{3}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{3\sqrt{5}}{5}$ ●

($\frac{3}{n}\to 0$, $\frac{4}{n}\to 0$, $\frac{2}{n^2}\to 0$, $\frac{2}{n}\to 0$, $\frac{1}{n^2}\to 0$)

7. $\lim\limits_{n\to+\infty}\left(\sqrt[3]{n+2}-\sqrt[3]{n-2}\right)$

Имеслой е ор облик $\infty-\infty$ шйо е неопределен израз. Цел е да го изйудиме $\sqrt[3]{}$ ор броййел.

Зайва дадемиой израз сакаме да го помножиме со $\left(\sqrt[3]{n+2}\right)^2+\sqrt[3]{n+2}\sqrt[3]{n-2}+\left(\sqrt[3]{n-2}\right)^2$ со цел да се примени ф-лайа

$$(A-B)\cdot(A^2+AB+B^2)=A^3-B^3$$

$A\to\sqrt[3]{n+2}$, $B\to\sqrt[3]{n-2}$

$$\lim_{n\to+\infty}\left(\sqrt[3]{n+2}-\sqrt[3]{n-2}\right)=\lim_{n\to\infty}\left(\sqrt[3]{n+2}-\sqrt[3]{n-2}\right)\cdot\frac{\left(\sqrt[3]{n+2}\right)^2+\sqrt[3]{n+2}\cdot\sqrt[3]{n-2}+\left(\sqrt[3]{n-2}\right)^2}{\left(\sqrt[3]{n+2}\right)^2+\sqrt[3]{n+2}\cdot\sqrt[3]{n-2}+\left(\sqrt[3]{n-2}\right)^2}$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt[3]{n+2}\right)^3-\left(\sqrt[3]{n-2}\right)^3}{\sqrt[3]{(n+2)^2}+\sqrt[3]{(n+2)(n-2)}+\sqrt[3]{(n-2)^2}}=$$

$$=\lim_{n\to\infty}\frac{n+2-n+2}{\sqrt[3]{(n+2)^2}+\sqrt[3]{(n+2)(n-2)}+\sqrt[3]{(n-2)^2}}=0$$

Кога $n\to\infty$ и $(n+2)^2\to\infty$; $\sqrt[3]{(n+2)^2}\to\infty$ ....

Те именийелой йежи кон ∞.

Имаме $\frac{4}{\infty}$, а йоа йежи кон 0 ●